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The equilibrium state of bifilar helix as element of metamaterials

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In the present article, we study the long bifilar helix in which electric currents are quasi-stationary, i.e. the wavelength of the electromagnetic field is much longer than the turn of the helix. All components of the force acting on a physically small element of one helix from the other helix having a big length are calculated. The case when the currents in the two helices have the same direction relative to the *x* axis is considered. The dependence of the radial component of the force of interaction between two helices on the pitch angle is determined. At various pitch angles the helices can attract and repel each other while the direction of the current does not change. It is found the value of pitch angle when two helices do not interact and bifilar helix, formed by them, is in equilibrium state.

1. Introduction

Metamaterials are artificially engineered and structured materials to have properties that have not yet been found in nature. The properties of metamaterials are derived both from the inherent properties of their constituent materials, as well as from the geometrical arrangement of those materials. They are made from assemblies of multiple elements fashioned from such materials as metals or plastics.

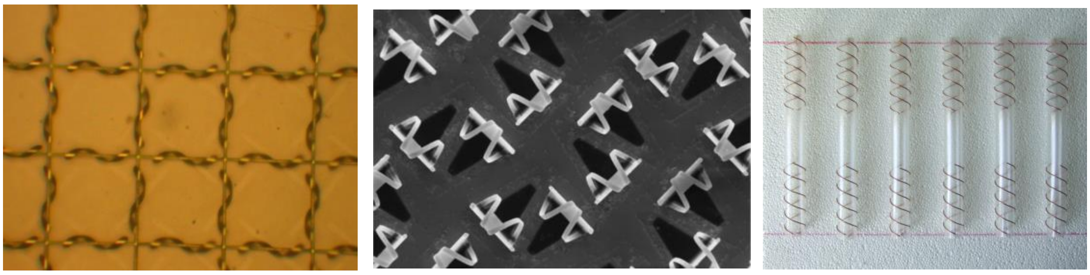
The metal helices are widely used as elements of metamaterials because at transmission of electrical current in them the electric dipole moments and magnetic moments are simultaneously generated. Consequently metamaterial based on helical elements displays as dielectric and magnetic properties, which enhances its use [1-11] (Fig.1).

A special place among the helices take bifilar helices consisting of two helical conductors. These conductors are arranged mutually symmetrically: the second helix is rotated with respect to the first helix on 180 degrees around a common helix axis (*x* axis). Electric currents in such bifilar helices are more balanced than in the case of single helices, which leads to the symmetry of properties of metamaterials: some components of the tensors of dielectric susceptibility and magnetic susceptibility vanish. At the same time, it raises the question about stability of the bifilar helix, since the electrical currents in it are close to each other and can interact strongly [12-15].

The objective of this article is the search of the value of pitch angle when two helices do not interact and bifilar helix, formed by them, is in equilibrium state. The determination of condition of equilibrium state can be used for design and manufacture of metamaterials consisting of bifilar helices as elements.All components of the force acting on a physically small element in the center of one helix from the other helix having a big length are calculated. The integral equation for determination of pitch angle is found and numerically solved. It is found the value of pitch angle when two helices do not interact and bifilar helix, formed by them, is in equilibrium state.

2. The equilibrium state of bifilar helix

In this work the long bifilar helix in which electric currents are quasi-stationary, i.e. the wavelength of the electromagnetic field is much longer than the turn of the helix is considered. In Fig.2 as a sample the two-turn helix and helix in a unfold form are presented. The geometry of the problem is presented in Fig.3.



(a) (b) (c)

Fig. 1. a) Photo of the helices array $In_{0.2}Ga_{0.8}As/GaAs/Ti/Au$ (a square grid on a photo is a negative photo resist from a polymeric material, thickness is about 1 micron) [4];
b) SEM image of an array of one-turn InGaAs/GaAs/Ti/Au helices resonant for THz range [5];
c) DNA-like helices resonant in microwave range [6-7].

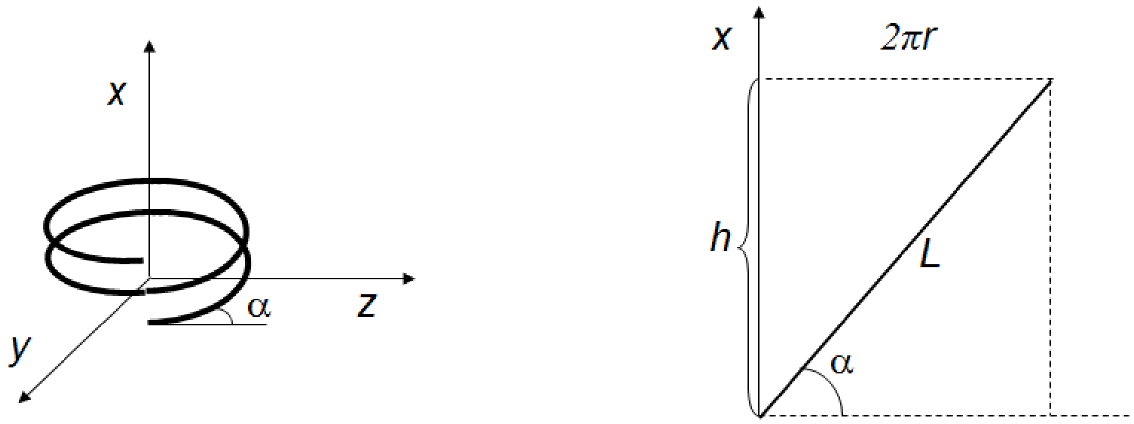
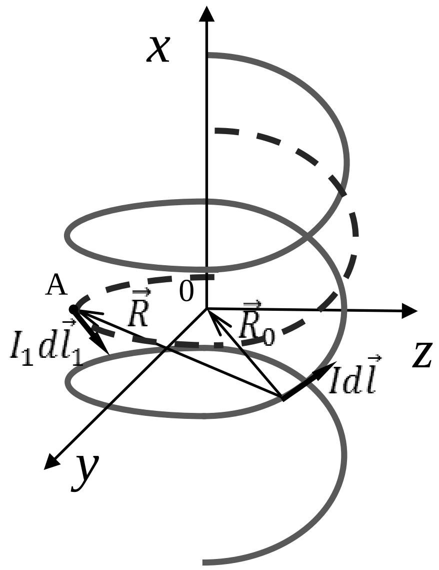


Fig. 2. Two-turn helix and helix in a unfold form, where r is the radius of the turn, L is the total wire length, α is the pitch angle, $h = 2\pi/|q|$ is the helix pitch, q is specific twisting of helix, and $q > 0$ for a right - handed helix and vice versa, $\cot\alpha = qr$.

The following notations are used: $\vec{R}_0$ is radius-vector from element of current $Id\vec{l}$ to the beginning of coordinate system, $I_1 d\vec{l}_1$ is element of current of second helix.
The relations between the projections of the vector in Cartesian and polar coordinate systems in our case are following

$$F_x = F_x$$
$$F_y = F_r \cos\varphi - F_\varphi \sin\varphi$$
$$F_z = F_r \sin\varphi - F_\varphi \cos\varphi$$

In the center of helix we obtaine for the point A: $\varphi = -\frac{\pi}{2} \Rightarrow \quad F_x = F_x\,;\; F_y = F_\varphi\,; F_z = -F_r\,.$



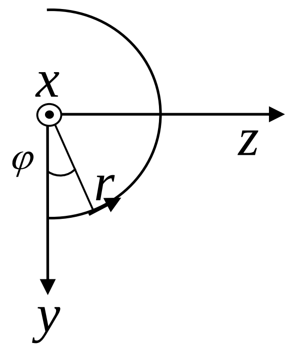


Fig. 3. The geometry of the problem (Cartesian and polar coordinate systems)

The vector $d\vec{B}$ of induction of a magnetic field at point A (Fig.3), generated by the elements of the current $Id\vec{l}$, calculated by the Biot-Savart formula

$$d\vec{B} = \frac{\mu_0}{4\pi}\frac{\left[Id\vec{l}\vec{R}\right]}{R^3}$$

Its components in Cartesian coordinate systems are following

$$B_x = \frac{\mu_0 Ir\cot\alpha}{2\pi}\int_0^{+\infty}\frac{1+\cos qx}{\left(x^2+2r^2(1+\cos qx)\right)^{3/2}}dx$$

$$B_y = \frac{\mu_0 I}{2\pi}\int_0^{+\infty}\frac{r\,(1+\cos qx)+\cot\alpha\cdot x\cdot\sin qx}{\left(x^2+2r^2(1+\cos qx)\right)^{3/2}}dx$$

$$B_z = \frac{\mu_0 I}{4\pi}\int_{-\infty}^{+\infty}\frac{r\sin qx-\cot\alpha\cdot x\cdot\cos qx}{\left(x^2+2r^2(1+\cos qx)\right)^{3/2}}dx = 0$$

since the integrand for B_z is an odd function. Thus $B_z = 0$ is a very important feature. It provides a balance of two symmetrically arranged helices, i.e. the absence of forces along the *x* and *y* axes in the center of helix (see below).
Suppose that at point A the element $d\vec{l}_1$ of second helix is located. The second helix is disposed symmetrically relative to the first helix. They form a bifilar helix. Then

$$dl_{1x} = dl_1\sin\alpha = dx_1$$
$$dl_{1y} = dl_1\cos\alpha = \frac{dx_1}{\sin\alpha}\cos\alpha = dx_1\cdot\cot\alpha$$
$$dl_{1z} = 0$$

We should note that $dl_{1x} > 0$ and $dl_x > 0$ because currents flow in the same direction with respect to the *x* –axis. dl_{1y} has opposite sign respect dl_y .The force acting on element of second helix $I_1 d\vec{l}_1$ calculated by the Ampere's formula

$$d\vec{F}_1 = \left[I_1 d\vec{l}_1 \vec{B}\right]$$

All components of the force $\vec{F}$ acting on a physically small element of one helix (Fig.1) from the other helix having a big length are calculated.

$$dF_{1x} = 0$$
$$dF_{1y} = 0$$
$$dF_{1z} = I_1 dx_1 \frac{\mu_0 I}{2\pi} \int_0^\infty \frac{r(1+\cos qx) + \cot\alpha \cdot x \cdot \sin qx}{\left(x^2 + 2r^2(1+\cos qx)\right)^{3/2}} dx - $$
$$- I_1 dx_1 \cot\alpha \frac{\mu_0 I r \cot\alpha}{2\pi} \int_0^\infty \frac{1+\cos qx}{\left(x^2 + 2r^2(1+\cos qx)\right)^{3/2}} dx$$

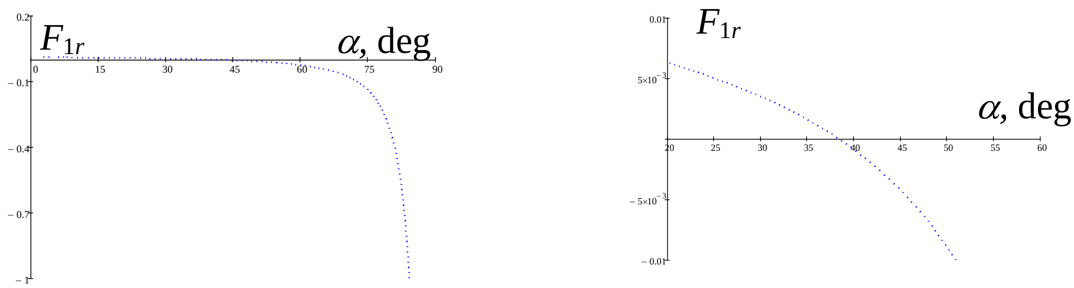
The force is calculated at the center of a long bifilar helix, i.e. when $x = 0$. We can see that the following components of force are equal to zero: $F_{1x} = 0$ and $F_{1\varphi} = 0$ where φ is the polar angle. Consequently, in the center of the helix the forces are absent that could rotate helices around their axis or move helices along this axis. At the same time the relation $F_{1r} \neq 0$ is satisfied, i.e. the component of force acting along the radius of the loop of helix r is present. We can see that the force F_{1r} depends strongly on the pitch angle α of bifilar helix.
The case when the currents in the two helices have the same direction relative to the x axis is considered. At the large pitch angles of helices the relation $F_{1r} < 0$ is satisfied, i.e. helices are mutually attracted. In the limiting case when $\alpha \to \pi/2, q = 0$ the well-known classical formula for the force of attraction of the same directed long parallel currents is obtained

$$\frac{dF_{1z}}{dx_1} = \frac{\mu_0 I I_1}{2\pi \cdot 2r}$$

Here *2r* is a distance between long parallel currents.
At small pitch angles of helices $\alpha = 0, q \to \infty$ when their loops are in the form close to the flat loops, the inequality $F_{1r} > 0$ is satisfied, i.e. helices repel each other.
We carry out the change of variables $u = qx$

$$dF_{1z} = 0 \Rightarrow F_{1r}(\alpha) = 0 \Rightarrow \int_0^\infty \frac{(1-\cot^2\alpha)(1+\cos u) + u\sin u}{\left(u^2 + 2\cot^2\alpha\,(1+\cos u)\right)^{3/2}} du = 0$$

We can see that the root of the equation $F_{1r}(\alpha) = 0$ is independent separately from the radius of the helix *r* and of the helix pitch $h = 2\pi/|q|$. The root of the equation $F_{1r}(\alpha) = 0$ is determined only by the pitch angle of helix α, and in this sense the angle $\alpha = arc\,\cot(qr)$ is a universal characteristic of the bifilar helix. The equation $F_{1r}(\alpha) = 0$ is numerically solved for the equilibrium bifilar helix and is shown that its root is α_0 (see Fig 4).



(a) (b)

Fig. 4. Normalized component of force F_{1r} versus pitch angle α: a) from 0 to 90 deg; b) from 20 to 50 deg

The equilibrium pitch angle α_0 is the same for all sizes of helices, both small and large, if the condition of quasi-stationary current $\lambda \gg P$ is satisfied. Here $P = \sqrt{(2\pi r)^2 + h^2}$ is the length of one turn of the helix, λ is wavelength of the electromagnetic field. The value of pitch angle $\alpha_0 = 38.4\ deg$ is found. At this value the relation $F_{1r}\,(\alpha) = 0$ is satisfied, i.e. two helices do not interact and bifilar helix, formed by them, is in equilibrium state.

3. Conclusion

We study the long bifilar helix in which electric currents are quasi-stationary. All components of the force acting on a physically small element in the center of one helix from the other helix having a big length are calculated. It is found the value of pitch angle $\alpha_0 = 38.4\ deg$ when two helices do not interact and bifilar helix, formed by them, is in equilibrium state. The determination of condition of equilibrium state can be used for design and manufacture of metamaterials consisting of bifilar helices as elements.

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